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On the reducibility of the new non-minimal sets of auxiliary fields for $N = 1$ supergravity

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Abstract. If a new criterion of reducibility for auxiliary field sets is employed, we show that the two new non-minimal sets for $N = 1$ supergravity are reducible.

Recently a technique was developed to allow the construction of off-shell supersymmetric theories (Rivelles and Taylor 1983). It is based on certain redefinitions of the fields (Rivelles and Taylor 1982a) belonging to the irreducible representations of the appropriate supersymmetry algebra. Mixing different irreducible representations and requiring these field redefinitions to produce the physical fields of the theory, plus a set of auxiliary fields, new off-shell theories were constructed for $N = 1$ (Rivelles and Taylor 1982b) and $N = 2$ (Rivelles and Taylor 1982c, see also Sohnius and West 1983) supergravity at the linearised level. It was also shown that for $N = 1$ there are two minimal sets, already known (Stelle and West 1978, Ferrara and van Nieuwenhuizen 1978, Sohnius and West 1981) and for $N = 2$ six minimal sets, two of them known earlier (de Wit and van Holten 1979, Fradkin and Vasiliev 1979, de Wit *et al* 1981). Finally, three sets were found in the non-minimal sector of the $N = 1$ theory, one of them being Breitenlohner's set (Breitenlohner 1977a, b), and all other sets (apart from these three) were shown to be reducible to either of the minimal forms. The full non-linear structure of the new theories was recently studied in Kugo and Uehara (1982a).

To show the irreducibility of the three non-minimal sets, a criterion of reducibility was adopted (Rivelles and Taylor 1983): a set is reducible if it is possible to redefine the fields in such a way that we get an irreducible set plus another set of fields, each closed among themselves under supersymmetry transformation, so that the second set can be put on-shell consistently without affecting the first one. As we shall see, this criterion is too strong and can be relaxed, so that the new non-minimal sets can be shown to be reducible whilst Breitenlohner's set remains irreducible.

All these theories can also be obtained from the $N = 1$ superconformal theory by appropriate choices of scale multiplets (Kugo and Uehara 1982b and references therein). However, van Proeyen and de Wit (Kugo 1982) pointed out that since all available scale multiplets have already been used to give the two minimal sets and Breitenlohner's set, the new non-minimal sets should be reducible. This, in fact, was demonstrated to be true by the use of field dependent gauge transformations which remove the extra scale multiplets (Kugo 1982).

This can be reconciled with our work if we adopt a broader criterion for reducibility: a set is reducible if it is possible to redefine the fields in such a way that we get an irreducible set plus another set whose fields do not need to close among themselves, but which can be proportional to those of the first set. This was not allowed before since the fields of each set should close among themselves and, therefore, should be a representation of the supersymmetry algebra. Such a redefinition corresponds to the field dependent gauge transformation mentioned above.

If we now apply this new criterion of reducibility to the first new non-minimal set (Rivelles and Taylor 1982b), and by setting $\lambda = 0$, we find (we use notations and conventions of Rivelles and Taylor (1982b))

$$a_\mu = (1-n)b_\mu \qquad f_1 = -(1-n)f_2 \qquad g_1 = -(1-n)g_2. \tag{1}$$

By consistency it follows that

$$\chi = -\tfrac{4}{3}(1-n)\mathrm{i}\boldsymbol{\gamma}\cdot\boldsymbol{R} \qquad f = \tfrac{4}{3}(1-n)(\partial^\mu\,\partial^\nu - \eta^{\mu\nu}\Box)h_{\mu\nu}. \tag{2}$$

If (1) and (2) are then substituted back into the supersymmetry transformation rules and Lagrangian, we get those of the minimal set (Stelle and West 1978, Ferrara and van Nieuwenhuizen 1978, Sohnius and West 1981). Similarly, for the second new non-minimal set (Rivelles and Taylor 1982b) $\lambda = 0$ implies

$$b_\mu = s = p = a = 0 \tag{3}$$

which by consistency requires

$$\chi = -\tfrac{4}{3}n(1-n)^{-1}\mathrm{i}\boldsymbol{\gamma}\cdot\boldsymbol{R} \qquad f = \tfrac{4}{3}n(1-n)^{-1}(\partial^\mu\,\partial^\nu - \eta^{\mu\nu}\Box)h_{\mu\nu} \tag{4}$$

and after substitution we find the new minimal set (Sohnius and West 1981). If we try to apply the same procedure to Breitenlohner's set we soon find $\boldsymbol{R}_\mu = 0$, demonstrating the irreducibility of this set.

Once more superconformal methods have helped the elucidation of some aspects of the Poincaré theory. The reducibility criterion showed to be more subtle than it looked at the beginning, and the new criterion, being more general than the old one, has now to be adopted.

So, we conclude that under this new reducibility criterion, there are only three irreducible sets for $N = 1$ Poincaré supergravity: the two minimal sets and Breitenlohner's set. The results for $N = 2$, of course, remain unchanged since all of them are minimal sets.

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References

Breitenlohner P 1977a *Phys. Lett.* **67B** 49
—— 1977b *Nucl. Phys.* B **124** 500
Ferrara S and van Nieuwenhuizen P 1978 *Phys. Lett.* **74B** 333
Fradkin E S and Vasiliev M A 1979 *Lett. Nuovo Cimento* **25** 79

Kugo T 1982 Private communication

Kugo T and Uehara S 1982a *Preprint Kyoto University, On the Non-Minimal Versions of N = 1 Poincaré Supergravity*

—— 1982b *Preprint Kyoto University, Conformal and Poincaré Tensor Calculii in N = 1 Supergravity*

Rivelles V O and Taylor J G 1982a *J. Phys. A: Math. Gen.* **15** 2819

—— 1982b *Phys. Lett.* **113B** 467

—— 1982c *Phys. Lett.* **119B** 111

—— 1983 *Nucl. Phys.* B **212** 173

Sohnius M and West P C 1981 *Phys. Lett.* **105B** 353

——— 1983 *Nucl. Phys.* B **216** 100

Stelle K and West P C 1978 *Phys. Lett.* **74B** 330

de Wit B and van Holten J W 1979 *Nucl. Phys.* B **155** 530

de Wit B, van Holten J W and van Proeyen A 1981 *Nucl. Phys.* B **184** 77